On the Behavior of Turbulent Boundary Layers near Separation

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Theme

ONE of the most important problems in aerodynamics is the phenomenon of separation that defines the initiation of the wake. The location of separation can be determined analytically by virtue of the boundary-layer theory, even though this theory is not valid in the immediate neighborhood of separation. In most practical applications the boundary layers are turbulent along the largest portion of the solid boundaries and most often, in the vicinity of separation. A large number of methods have been developed to solve the turbulent boundary-layer equations with an appropriate closure assumption, but a straightforward numerical integration in their differential form was attempted only in the late sixties. A number of such investigations is referenced in the accompanying engineering report of this paper.[1]

For steady laminar and turbulent boundary-layer flow the location of separation is signaled by the vanishing of the wall shear. This criterion has been extensively used in laminar flows for both experimental and analytical investigations.[2] However, for turbulent flow the use of simplified theoretical models has forced investigators to develop heuristic criteria based on approximate expressions that approach certain values in the neighborhood of separation.

The present report reconsiders the problem of turbulent separation. The classical closure assumptions are restated and their applicability in the neighborhood of separation is discussed. Some typical features of the flow are checked against available experimental data and previous theoretical predictions. Further, the location of separation is calculated and the response of the boundary-layer equations in the neighborhood of separation is carefully investigated. The present effort is intended to be a step towards the generalization of the method to unsteady flow.

Contents

The turbulent boundary layer equations in their averaged form, for steady incompressible flow, read

$$u\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad -\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}x}+\frac{\partial}{\partial y}\left(\frac{\mu}{\rho}\frac{\partial u}{\partial y}-\overline{u'v'}\right)$$

$$-\frac{\partial}{\partial x}\left(\overline{u'^2}-\overline{v'^2}\right)$$

where x, y, and u, v are the coordinates along and perpendicular to the wall and the average velocity components in these directions respectively, while u',v' are the corresponding instantaneous velocity fluctuations; further, ρ is the density, μ is the viscosity, p is the pressure, and an overbar denotes time averaging. The terms involving averaging of random fluctuating quantities are not determined at this stage and require some approximate model in order to close the system of equations. The terms $\partial/\partial x(\bar{u}'^2-\bar{v}'^2)$ have been proved experimentally to be very small compared to the term $\partial/\partial y\ (\bar{u}'\bar{v}')$ and are usually dropped from the momentum equation. This is not true in the neighborhood of separation, as recent experiments have disclosed and the present investigation includes some discussion on this point. A two layer eddy viscosity model was used based on the Van Driest and Clauser models and the final development of Cebeci.[3] With the term $\partial/\partial y\ (\bar{u}'\bar{v}')$ so replaced the previous system of equations was solved numerically according to an implicit finite-difference scheme.[1] Except for a few refinements, as for example an iteration scheme to update the coefficients of the nonlinear equations, the present method is similar to other methods developed in the last few years (see references in the accompanying report). Calculations were checked against previous analytical results as well as various experimental data.[1]

The main goal of this effort was the study the behavior of the turbulent boundary-layer equations near separation. Unfortunately there is very little experimental information available and most of it is outdated. Cebeci et al[4] have already collected most of the experimental data on turbulent separation as well as the available methods for analytical prediction of separation and presented an exhaustive comparison. We have repeated some of the calculations only as test cases, and we proceeded further to examine the detailed features of this mathematical model in the neighborhood of separation. Figure 1 shows a detail of the outer flow distribution of the experiments of Schubauer and Klebanoff[5] as well as a comparison of predictions of separation by the present and other methods.[6-8] The reader should note that all methods are approximate and based on heuristic assumptions except for the methods of Refs. 1 and 4, which integrate the full form of the differential equations. It is observed that integration of the equations in their differential form appears to be more accurate than any of the other methods.

It was found that a singularity appears at the point of zero skin friction and convergence of the numerical scheme is not possible at this station. A more careful examination revealed that as the station of separation is approached, quantities like the normal component of the velocity v or streamwise space derivatives, $\partial/\partial x$, start growing sharply. In fact it appears that the growth rate is proportional to the inverse square root of the distance from separation in accordance with Goldstein's theoretical model for laminar flow.[2] This is shown in Fig. 2 where the variation of the square of the skin friction coefficient appears to be very nearly linear. The above findings are rather significant in view of the fact that the singular behavior is more reliable as a criterion for separation than the vanishing of skin friction. This was demonstrated theoretically and experimentally for steady laminar flows over

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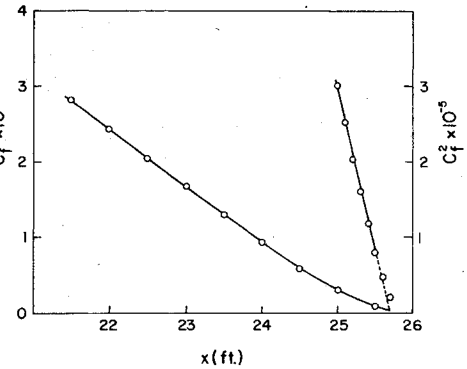

Fig. 1 Predictions of location of separation for the outer flow distribution of Ref. 5 according to the methods of: 1) Ref. 6; 2) Ref. 8 (2 parameters); 3) Ref. 8 (1 parameter); 4) Ref. 7; 5) experimental, Ref. 5; 6) Ref. 3 and present method.

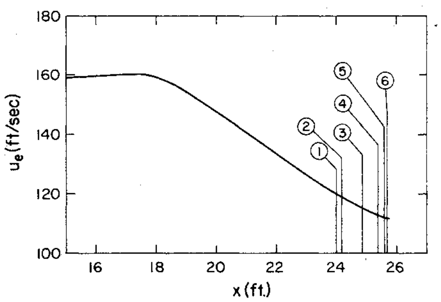

Fig. 2 The square of the skin friction coefficient c_f^2 calculated with two different mesh sizes for the outer flow distribution of Ref. 5.

moving walls[9] and for unsteady laminar flows over fixed walls.[10]

Of particular interest are also the experimental estimations[11] of the terms $\partial/\partial x\ (\bar{u}'^2)$ and $\partial/\partial y\ (\bar{u}'\bar{v}')$ in the neighborhood of separation. The measurements show that even though the first term, traditionally assumed to be negligible, grows substantially as separation is approached, the convective terms, $u\partial u/\partial x + v\partial u/\partial y$, eventually outgrow all other terms of the momentum equation in this region. As a result the importance of a correct modeling of such terms is diminished. It is intriguing to note that the singular behavior discovered by the present analysis predicts exactly the same behavior. The terms $u\partial u/\partial x$ and $v\partial u/\partial y$ are of order $x_l^{-1/2}$ and therefore grow fast as $x_l \to 0$, where x_l is the distance from separation, while all other terms of the momentum equation remain finite. Therefore, the boundary-layer theory predicts correctly the trend but exaggerates the behavior and allows the convective terms to grow without bounds thus forcing the termination of the calculations. It is intriguing to note that the separation singularity is present even if the boundary-layer equations are solved by another method. For example, Nash, Carr and Singleton[12] employ the turbulent energy equation which renders the system hyperbolic, but retain the nonlinear terms of the boundary-layer equations while Kuhn and Nielsen[13] use an integral method. These investigators have found that the singular character of the equations in the neighborhood of separation is still present.

At this point we can make some comparisons with the work of Refs. 14 and 15. In these references the authors examine experimentally and numerically, respectively, the phenomenon of laminar separation with forced harmonic outer flow oscillation, which is intimately related to turbulent separation. Despard and Miller[14] conclude that separation should occur at a station where the wall shear is negative throughout the cycle of oscillation and has zero as its maximum value. This clearly is not the case in turbulent flow where the experiments indicate that at the station of separation the wall shear oscillates between positive and negative values but vanishes in the average.[11] In some cases Tsahalis and Telionis,[15] investigating forced oscillations of laminar flows, arrived at an approximately similar conclusion, consistent with the findings of experiments on turbulent flows. Namely that separation is located somewhere in between the extreme locations of the excursions of zero skin friction.

The results of the present calculations appear to agree with the experimental data, al least as far as the location of separation is concerned. This is not brought here as yet another proof that turbulent separation occurs at the point of zero averaged wall shear. Unfortunately, the method fails to predict other features of the flow at separation, as for example, velocity profiles. This indicates that mixing length models may be inadequate in the neighborhood of separation. Finally we speculate that the mechanism of turbulent separation must be very similar to that of laminar separation, since in both cases it is the convective terms that grow sharply as separation is approached. Turbulent flow perhaps affects the location of separation only by influencing its development from the leading edge or the stagnation point.

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